EE 233 Circuit Theory
Lab 4: Filters

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1. Introduction

The objectives of this lab are to design filters from a given topology and specifications, analyze the characteristics of the designed filters, measure the characteristics of the designed filter, and build up the entire mixing console.

Filter design is the process of designing a signal processing filter that satisfies a set of requirements. The purpose of the design is to develop a type of filter that meets each of the requirements to a sufficient degree in order to make it useful.

The filters that are built in this lab are part of an equalizer system. Connect these filters to the buffer and output summing amplifier and you will have a three-channel equalizer. The equalizer is used to alter the frequency response of the audio system and adjust the amplitude of audio signals at particular frequencies. The audio mixing will be completed when the equalizer is built. Please read Supplemental Material Audio Mixer for more information of the whole project.

2. Precautions

None of the devices used in this set of experiment are particularly static sensitive; nevertheless, you should pay close attention to the circuit connections and to the polarity of the power supplies, operational amplifier and oscilloscope inputs.

3. Prelab

In this procedure, you are going to calculate the transfer function for a given filter topology and compare it with a SPICE simulation. This process will help you understand how this filter works. The calculation is relatively complex, and this procedure will help you to get the transfer function progressively.

3.1 Simple Filter

1. Show the transfer function $H(s) = V_{out}(s)/V_{in}(s) = -\left[\frac{1}{Z_3} + \frac{Z_5}{Z_1} \left(\frac{1}{Z_3} + \frac{1}{Z_4} + \frac{1}{Z_5}\right)\right] \left[\frac{1}{Z_4} + \frac{Z_5}{Z_2} \left(\frac{1}{Z_3} + \frac{1}{Z_4} + \frac{1}{Z_5}\right)\right]$ in s-domain for the filter in Figure 1.

![Figure 1 Simple filter](image-url)
2. Assume $Z_1 = Z_2 = Z$. Rewrite the transfer function and design the other impedances so that the magnitude of the transfer function is always $|H(s)| = 1$.

This topology is quite flexible because it allows the filter to be a low-pass, high-pass, or even band-pass or band-stop filter with a single potentiometer. Now consider this filter in the frequency domain, and the following questions will help you to figure out some characteristics of this filter.

3. Assume all the impedances except $Z_5$ are resistors, and that $Z_5$ is a capacitor. Assume $Z_1 = R_1$, $Z_2 = R_2$, $Z_3 = R_3$, $Z_4 = R_4$ and $Z_5 = \frac{1}{j\omega C}$. Derive the frequency response with $R_1$, $R_2$, $R_3$, $R_4$ and $C$. Then using $R_1 = R_2 = 2.4 \text{k}\Omega, C = 0.01 \mu\text{F}, R_3 = 100 \text{k}\Omega, R_4 = 50 \text{k}\Omega$, draw the gain of the filter from 10 Hz to 5 kHz and prove that the filter is a low-pass filter. After that, switch $R_3$ and $R_4$ and draw the gain from 10 Hz to 5 kHz and prove that the filter is a high-pass filter.

4. Now suppose we choose the impedances $Z_1, Z_2, Z_3, Z_4$ and $Z_5$ carefully and it becomes a band-pass filter. Assume $Z_1 = Z_2$, and if $Z_3$ and $Z_4$ are switched is it a band-pass or band-stop filter?

**Hint:** Switch $Z_3$ and $Z_4$ in the transfer function and compare the new one with the original transfer function.

### 3.2 Band-pass and Band-stop Filters

The topology of the filter that is going to be implemented in the mixing console is similar to, but more complex than, the filter that has been calculated. Now we are going to analyze the filter that is going to be implemented in the audio mixer. The goal is to make three band-pass filters with different central frequencies and a tunable gain so that we can change the gain of the equalizer at separate frequency ranges. Once the band-pass filter is made the band-stop filter can be achieved by changing the potentiometer in each filter, which means that the same filter can be changed between band-pass and band-stop by only tuning the potentiometer.

1. Read Extra Credit Y-Δ Transform at the end of the prelab. Use the transform equations and change the Δ topology among $N_1, N_2$ and $N_3$ in Figure 2 to Y topology and label the new impedance as $Z_a, Z_b$ and $Z_c$. Draw the new circuit and turn it in with your completed prelab. Derive the expressions for new impedances in your topology with $C_1$ and $R_5$ in the s-domain.

**Hint:** Node $N_3$ is on the potentiometer $R_5$, which means the movable terminal of the potentiometer is at the node $N_3$. You could assume that the resistance on the left of the movable terminal is $\gamma R_5$, where $0 \leq \gamma \leq 1$, while the resistance on the right is $(1 - \gamma)R_5$. 

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2. Derive the transfer function for the filter with \( R_1, R_2, R_3, R_4, Z_a, Z_b, Z_c, C_1 \) and \( C_2 \).

**Hint:** You could use the equation derived in 3.1 item 1.

3. Assume \( R_1 = R_2 = 240 \, \text{k}\Omega, R_3 = R_4 = 2.4 \, \text{k}\Omega \) and \( R_5 = 100 \, \text{k}\Omega \). Derive the transfer function, then choose capacitors in the lab kit to make three adjustable band-pass or band-stop filters whose center frequencies are centered around 250 Hz, 1 kHz, and 4 kHz.

**Hint:** The AC analysis in Multisim could help in finding the center frequency. Also, you could use the Matlab code in Reference 5.1: Matlab Program for Center Frequency to achieve the center frequencies. The Matlab program is written with the equations you have derived in the questions above.

4. Use AC analysis in SPICE to simulate the gain of these three filters. Record the center frequencies, cutoff frequencies, and gain and turn them in with your completed prelab.

**Extra Credit Y-Δ Transform**

A Y-Δ transformation should be used to analyze the filter. The Y-Δ transform, also written wye-delta, is a mathematical technique to simplify the analysis of an electrical network. The name derives from the shapes of the circuit diagrams, which look like the letter Y and the Greek capital letter Δ, respectively. These shapes are shown in Figure 3.
1. Show that the following equations for transformation from a Y-load to a Δ-load circuit are correct.

\[
Z_a = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_1} \\
Z_b = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_2} \\
Z_c = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_3}
\]

**Hint:** The Y-load and Δ-load can be transformed into each other because they are assumed to be equivalent. This equivalence means that for any external voltages \((V_1, V_2\) and \(V_3\)) applying at the three nodes \((N_1, N_2\) and \(N_3\)), the corresponding currents \((I_1, I_2\) and \(I_3\)) are exactly the same for both the Y and Δ circuit, and vice versa.

2. Show that the following equations for transformation from Δ-load to Y-load circuit are correct.

\[
Z_1 = \frac{Z_b Z_c}{Z_a + Z_b + Z_c} \\
Z_2 = \frac{Z_a Z_c}{Z_a + Z_b + Z_c} \\
Z_3 = \frac{Z_a Z_b}{Z_a + Z_b + Z_c}
\]

4. **Experimental Procedure and Data Analysis**

4.1 **Band-pass and Band-stop Filters**

1. Build the circuit in Figure 2 using power supplies of ±12 V and the components from your design in the Prelab section. The center frequencies are designed for approximately 1 kHz.

**Question 1:** What are the resistors and capacitors that you used in this filter?
Before doing the following experiments with spectrum analyzer, it is better to check your circuit with function generator and oscilloscope and make sure that the circuits are working well as band-pass or band-reject filters.

2. Plot the gain of the filter between 10 Hz and 5 kHz by setting the potentiometers to 25%, 50% and 75%. There are two ways to plot the gain. The first is to set the function generator to output a sine wave input with a small amplitude so that the output is not affected by the slew rate and saturation of the op-amp during this part. Starting with an input frequency of 10 Hz, then varying it using the 1-2-5 sequence up to 5 kHz (i.e. set input frequency to 10 Hz, 20 Hz, 50 Hz, 100 Hz, 200 Hz, ..., up to 50 kHz). Display the input and output waveforms (2-3 complete cycles) on the scope. For each frequency setting above, measure the gain of the three circuits. Alternatively, use of the spectrum analyzer could automatically measure the gain. Please read Reference 5.2: Spectrum Analyzer to learn the operations of the spectrum analyzer.

**Question 2:** Turn in the hard copy of the gain with your lab report. Use a table to store the center frequency, 3-dB frequencies, and maximum gain for the filter by setting the potentiometers to 25%, 50% and 75%.

3. Build the other two filters whose center frequencies are designed at 250 Hz and 4 kHz. Repeat step 4.1 item 2 and 3 for both of these two filters.

**Question 3:** What are the resistors and capacitors that you used in these filters? Turn in the hard copy of the gain with your lab report. Use a table to store the center frequency, 3-dB frequencies, and maximum gain for the filter by setting the potentiometers to 25%, 50% and 75%.

**4.2 Audio Mixer**

Now you should be ready to build the whole mixing console and play music with it. Combine all of the parts that you have built in the correct order and use the following steps to test your circuit system.

1. Use a sine wave with 250 Hz and a small amplitude as an input to the system. Make sure that there is no saturation among the signal processing in the system.

**Question 4:** Display the input and output waveforms in 2-3 complete cycles on the scope and turn in the hard copy with your lab report.

2. Change the potentiometers in the equalizers and display the output on the oscilloscope.

**Question 5:** Which filter controls the amplitude of the output filter? Turn in hard copies of the scope to support your conclusion.

3. Use sine waves with 1 kHz and 4 kHz and repeat 4.2 item 1 and 2.

**Question 6:** Which filter controls the amplitude of the output filter at 1 kHz and 4 kHz? Turn in hard copies of the scope to support your conclusion.

4. Plot the gain of the audio mixer between 10 Hz and 5 kHz with a single input. There are two options to do this task. The first is to use function generator and a sine wave input with small amplitude so that the output is not affected by the slew rate and saturation in this part. Then start with an input frequency.
of 10 Hz and vary it using 1-2-5 sequence up to 5 kHz (i.e. set input frequency to 10 Hz, 20 Hz, 50 Hz, 100 Hz, 200 Hz, … up to 50 kHz). Display the input and output waveforms (2-3 complete cycles) on the scope. For each frequency setting above, measure the gain of the three circuits. The second option is to use the spectrum analyzer to automatically measure the gain. Please read Reference 5.2 Spectrum Analyzer to learn the operations of spectrum analyzer.

**Question 7:** Turn in the plot of the gain in your lab report and comment on how many bands you see in the plot.

### 4.3 Lab Test Preparation

If everything is working well and appears as you expected, use music or instruments as input signals and play them with the audio mixer system. Let your TA check your circuits to make sure it is working well, because it will be used for the demo in your lab test.

### Extra Credit

1. If you have not completed the microphone circuit in Lab 3, you could still work on it. You will still get the extra credit when you show it to your TA and answer the question in your lab report.

2. Build the whole audio mixer system in SPICE and use the AC analysis to see the transfer function of the whole system. Show your simulation to your TA and keep it for the lab test.

**Question E:** What does the transfer function look like with only one input track? Describe the roles of each part in the system.

### 5. Reference

#### 5.1 Matlab Program for Center Frequency

```matlab
% EE233 Lab 4 Matlab code
% Last date of updating: May 14, 2014
% This program is used for calculate center frequencies in EE233 Lab 4
% All parameters (R1, R2, R3, R4, R5, gama, minfreq, maxfreq) could be set
% manually in this program. One of C1 and C2 could be a vector for
% sweeping.
% Parameters:
% R1, R2, R3, R4, R5 and gama are the parameters corresponding to Figure 2
% in Lab 4
% minfreq and maxfreq are the minimum and maximum frequency in frequency
% domain analysis
% Units:
% resistor: ohm
% capacitor: Farad
% frequency: Hz

clear; clc; clf; close all;
% Set parameters
R1 = 240e3;       % R1 = 240k ohm
R2 = 240e3;       % R2 = 240k ohm
```
R3 = 2.4e3;  % R3 = 2.4k ohm
R4 = 2.4e3;  % R4 = 2.4k ohm
R5 = 100e3;  % R5 = 100k ohm
gama = 0.25;  % gama = 0.25
minfreq = 10;  % frequency starts at 10 Hz
maxfreq = 1e6;  % frequency ends at 5 kHz
% Set C1 and C2
C1 = 0.1e-6;  % set C1 here
% C2 = linspace(0.001e-6,0.1e-6,1e2);  % set C2 here
C2 = 0.01e-6;

if(length(C1) == 1 && length(C2) == 1)  % C1 and C2 are both numbers
    [f0, G0, fc1, fc2] = Lab4_filter_Gain(R1, R2, R3, R4, R5, C1, C2,...
        gama, minfreq, maxfreq, 'SingleCalculation');
    fprintf('Center frequency f0 = %6g Hz\n', f0);
    fprintf('Gain at f0 is G0 = %6g \n', G0);
    fprintf('Cutoff frequencies: fc1 = %6g Hz, fc2 = %6g Hz\n', fc1, fc2);
elseif (length(C1) > 1 && length(C2) == 1)  % C1 is a vector
    f0 = zeros(1, length(C1));
    G0 = zeros(1, length(C1));
    for m = 1: length(C1)
        [f0(m), G0(m), fc1, fc2] = Lab4_filter_Gain(R1, R2, R3, R4, R5, C1(m), C2,...
            gama, minfreq, maxfreq, 'VectorCalculation');
    end
    plot(C1, f0);
    xlabel('C1 (Farad)');
    ylabel('Center frequency (Hz)');
    grid on;
    hold on;
elseif (length(C1) == 1 && length(C2) > 1)  % C2 is a vector
    f0 = zeros(1, length(C2));
    G0 = zeros(1, length(C2));
    for m = 1: length(C2)
        [f0(m), G0(m), fc1, fc2] = Lab4_filter_Gain(R1, R2, R3, R4, R5, C1, C2(m),...
            gama, minfreq, maxfreq, 'VectorCalculation');
    end
    plot(C2, f0);
    xlabel('C2 (Farad)');
    ylabel('Center frequency (Hz)');
    grid on;
    hold on;
else
    err = MException('Input:TooMuchInput',...
        'Too much input.\nOnly one parameters could be swept.');
    throw(err);
end

% Matlab sub-function for EE233 Lab 4 Filters
% This function is used to find
%     center frequency, f0
%     gain at center frequency, G0
% cutoff frequencies, fc1, fc2
% plot gain in frequency domain
% Given by
% Resistance (R1,R2,R3,R4,R5)
% Capacitance (C1,C5)
% Left portion of potentiometer (gama)
% frequency range
% Command: SingleCalculation, VectorCalculation
%
% IF R1 = R2 and R3 = R4
% If gama > 0.5, the gain is always less than 1, so it is a band-stop filter
% If gama < 0.5, the gain is always greater than 1, so it is a band-pass filter
% If gama = 0.5, the gain is always equal to 1
%
% Notice: Input references should all be considered as numbers

function [fc, Gc, fc1, fc2] = Lab4_filter_Gain(R1, R2, R3, R4, R5, C1, ... 
                                        C2, gama, minfreq, maxfreq, command)
    % angular frequency range
    samples = 1e6;
    freq = linspace(minfreq, maxfreq, samples);
    w = 2 * pi * freq;

    % s domain in steady state
    s = 1i * w;

    % delta-Y transformation
    Za = gama .* R5 ./ (1 + s * R5 * C1);
    Zb = (1 - gama) * R5 ./ (1 + s * R5 * C1);
    Zc = gama * (1 - gama) * R5 ^2 * C1 * s ./ (1 + s * R5 * C1);

    % transfer function
    Hs = (1 ./ (R3 + Za) + (Zc + 1 ./ (s .* C2)) ./ R1 .* ... 
         (1 ./ (R3 + Za) + 1./ (R4 + Zb) + 1./ (Zc + 1 ./ (s .* C2)))) ./ ... 
         (1 ./ (R4 + Zb) + (Zc + 1 ./ (s .* C2)) ./ R2 .* ... 
         (1 ./ (R3 + Za) + 1./ (R4 + Zb) + 1./ (Zc + 1 ./ (s .* C2)));

    % Gain and its plot
    gain = abs(Hs);
    if(strcmp(command, 'SingleCalculation'))
        clf;
        loglog(freq, gain);
        hold on;
        grid on;
        xlabel('frequency (Hz)');
        ylabel('gain');
    end

    % find the range of gain
    max_gain = max(gain);
    min_gain = min(gain);
% find the center frequency and its gain
if (gama > 0.5) % band-stop filter
    fc = freq(find(gain == min_gain, 1, 'first'));
    Gc = min_gain;
elseif (gama < 0.5) % band-pass filter
    fc = freq(find(gain == max_gain, 1, 'first'));
    Gc = max_gain;
else % Hs = 1
    err = MException ('Gain:ConstantGain', ...
                      'The gain is always 1 and the center frequency cannot be identified. Please set gama not equal to 0.5');
    throw(err);
end

% find the cutoff frequencies
if(strcmp(command, 'SingleCalculation'))
    if (((gama > 0.5 && max_gain >= (1 / sqrt(2))) || (gama < 0.5 && max_gain <= sqrt(2)))
        errBound = MException('CutoffFrequency:NoCutoffFrequency', ...
                               'The gain is too small that there is no cutoff frequecies for this filter. Please change to other options.');
        throw(errBound);
    else
        cutoff_gain = max_gain / sqrt(2);
        if (gama > 0.5) % band-stop filter
            fc1 = freq(find(gain <= cutoff_gain, 1, 'first'));
            fc2 = freq(find(gain <= cutoff_gain, 1, 'last'));
        else % band-pass filter
            fc1 = freq(find(gain >= cutoff_gain, 1, 'first'));
            fc2 = freq(find(gain >= cutoff_gain, 1, 'last'));
        end
        if (isempty(fc1) || isempty(fc2) || fc1 == fc2 || fc1 == 0 || fc2 == maxfreq)
            errBound = MException('CutoffFrequency:OutOfRange', ...
                                   'The frequency range is not enough to cover two cutoff frequencies. Please increase your frequency range');
            throw(errBound);
        end
    end
else
    fc1 = 0;
    fc2 = 0;
end

% title on the plot
str = sprintf('Center frequency f0 = %2f Hz
              Gain at f0 is G0 = %2f
              Cutoff frequencies fc1 = %2f Hz, fc2 = %2f Hz', ...
                fc, Gc, fc1, fc2);
    title(str);
5.2 Spectrum Analyzer
The spectrum analyzer located in the lab is an HP 3562A Dynamic Signal Analyzer. It is used to measure
the magnitude of an input signal versus the frequency within the full frequency range of the instrument.
Its primary use is to measure the power of the spectrum of known and unknown signals. Use the
following steps to measure the frequency response of your circuit system:

1. Connect **SOURCE** to the input of your circuit and **CHANNEL 1** to the output of the circuit.
2. Press **MEAS MODE** button in the MEASUREMENT panel.
3. There are several selections to choose from on the right column of the screen. Choose **SWEPT SINE**
   by pressing the button on the right of this item.
4. Choose **LOG SWEEP** on the right column of the screen.
5. Press **FREQ** button in the MEASUREMENT panel.
6. Choose **START FREQ** and set it to 10 Hz using the digital buttons in the ENTRY panel. The value
   will be shown at the bottom line of the screen.
7. Choose **STOP FREQ** and set it to 5 kHz.
8. Choose **Sweep Rate**. The sweeping is more accurate if the sweep rate is larger, but it will take
   longer to run. Change the sweep rate depending on whether a faster or more accurate sweeping is
   desired.
9. Press **SOURCE** button in the MEASUREMENT panel.
10. Choose **SOURCE LEVEL** and set it to 1 Vpk.
11. Choose **SOURCE ON** and **Sweep Up**.
12. Press **MEAS Disp** in the SELECT DATA panel and choose **POWER SPEC** on the right column of
    the screen. If you wish to go back to the state menu, you could press **STATE/TRACE** in the
    SELECT DATA panel.
13. Press the **SCALE** button in the DEFINE TRACE panel and choose **Y AUTO SCALE**.
14. Press the **START** button in the CONTROL panel and observe the output on the screen.